A Comparison of HRV Techniques: The Lomb Periodogram *versus* The Smoothed Pseudo Wigner-Ville Distribution

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1. INTRODUCTION

After outlining the theory behind the author's approach to tachograms and spectral analysis, HRV calculated using the Lomb periodogram will be compared to HRV determined via time-frequency analysis. This report will seem brief in parts; this was done to keep the page count low.

2. FUNDAMENTALS

2.1 Tachograms

The purpose of a tachogram (from the Greek *tachos*, speed) is to quantify changes in heart rate over short time periods, ordinarily less than one day. Typically, the heart's rate of contraction is sampled once per beat (and hence unevenly in time), being taken as the inverse of the interval which occurs between consecutive QRS complexes (known as an RR interval). Since the Fourier transform algorithm requires even sampling, the tachogram is often re-sampled using interpolation. However, this approach introduces new problems: for example, linear interpolation is a poor approximation, and cubic splines create unacceptable oscillations when one RR interval is unusually longer than its predecessor. Perhaps more importantly for cross-correlation with other signals, such as instantaneous blood pressure, the process of assigning the RR interval to one end of that interval (either at the beginning or end) creates a tachogram out of phase with the true heart rate.

There does exist a logical alternative to uneven sampling followed by interpolation. Berger *et al.* [1] developed a beautifully simple algorithm which samples the heart rate directly but evenly, so no later interpolation is required (see Figure 1). As depicted in Figures 2 and 3, this algorithm surpasses cubic spline interpolation on the issues of spurious oscillations and incorrect phase. This is the method employed by the author to prepare tachograms for real patient data, the subject of a future report.

A second strategy to avoid the drawbacks of interpolation is to compute the Fourier spectrum directly from the unevenly sampled tachogram. The Lomb periodogram is an excellent candidate for this operation, since it weights the data on a point-by-point basis rather than on a per-interval basis. It has been shown that the Lomb periodogram can provide a more accurate estimate of a tachogram's power spectral density (PSD) than interpolation followed by a regular Fourier transform [2]. However, the author is not aware of any effort to combine this valuable method with time-frequency analysis (see Section 2.2).

2.2 Obtaining a Spectrum Using Time-Frequency Analysis

The researcher processing non-stationary signals has many time-frequency methods from which to choose. For example, developed for quantum mechanics by Wigner [3] and introduced to signal processing fifteen years later by Ville [4], the Wigner-Ville distribution is defined by:

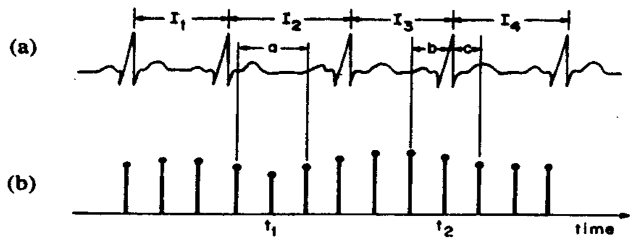

*Figure 1: **Berger*** **et al.***'s algorithm.* ***(a)*** *A segment of an ECG (electrocardiogram) signal.* ***(b)*** *The heart rate samples corresponding to the ECG signal in (a), determined using Berger* et al*'s algorithm. The fraction (often less than unity) of RR intervals within the local window centred at t_1 is a/I_2, and at t_2 is $b/I_3 + c/I_4$. The value of the heart rate at each sample point is taken to be the number of intervals that fell within the local window centred at that point divided by the width of the window. This calculation is performed at each point in (b), i.e., four more times between t_1 and t_2. [1]*

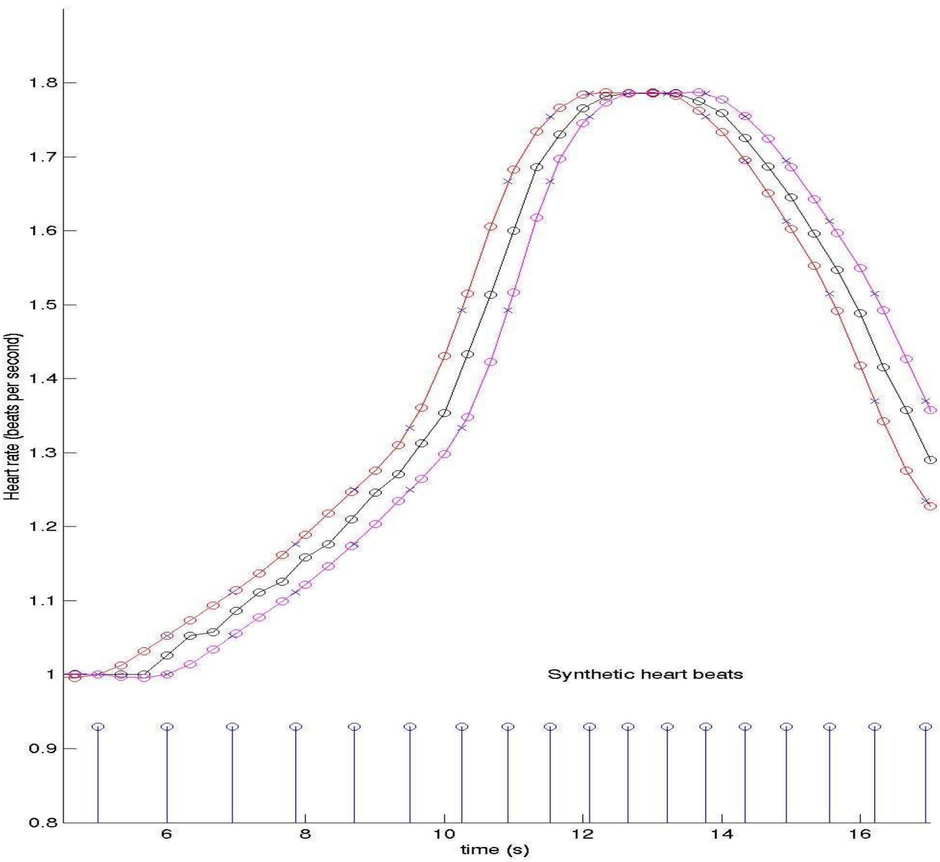

Figure 2: Heart rate phase. *(Synthetic data.) Berger* et al.*'s algorithm produces a tachogram (black) in phase with heart rate. Cubic splines (red and magenta) will always be out of phase, owing to the limitations of the initial uneven sampling. (In uneven sampling, "RR-intervals" are assigned at the points where heart beats occur, and the values of these intervals are marked as blue x's in the diagram .) It is tempting to consider averaging the two cubic splines results, which would together produce a smooth plot in phase with heart rate – provided the problem identified in Figure 3 could be conquered.*

$$W(t,\omega) = \int_{-\infty}^{\infty} f(t+\frac{\tau}{2}) f^*(t+\frac{\tau}{2}) e^{-i\omega\tau} d\tau$$

where f^* denotes complex conjugation. This equation provides a representation of a function f(t) in the joint time-frequency domain, and is indeed one of the most popular time-frequency analysis methods for biological signals. Discussion of alternatives will not be covered in this document.

The above equation is seldom used in its pure form, and the Smoothed Pseudo Wigner-Ville Distribution (SPWVD) is employed instead. The *discrete* form of the SPWVD is given as:

$$W(n,m) = \tfrac{1}{2} N \sum_{k=-N+1}^{N+1} |h(k)|^2 \sum_{p=-M+1}^{M-1} g(p)\, z(n+p+k)\, z^*(n+p-k)\, e^{\frac{-2i\pi k m}{M}}$$

Here, h(k) and g(p) represent frequency and time smoothing, respectively. The advantage of smoothing is the removal of spurious cross-terms; in fact, spectral interference is so high with the pure Wigner-Ville distribution that it precludes implementation in most real applications [5].

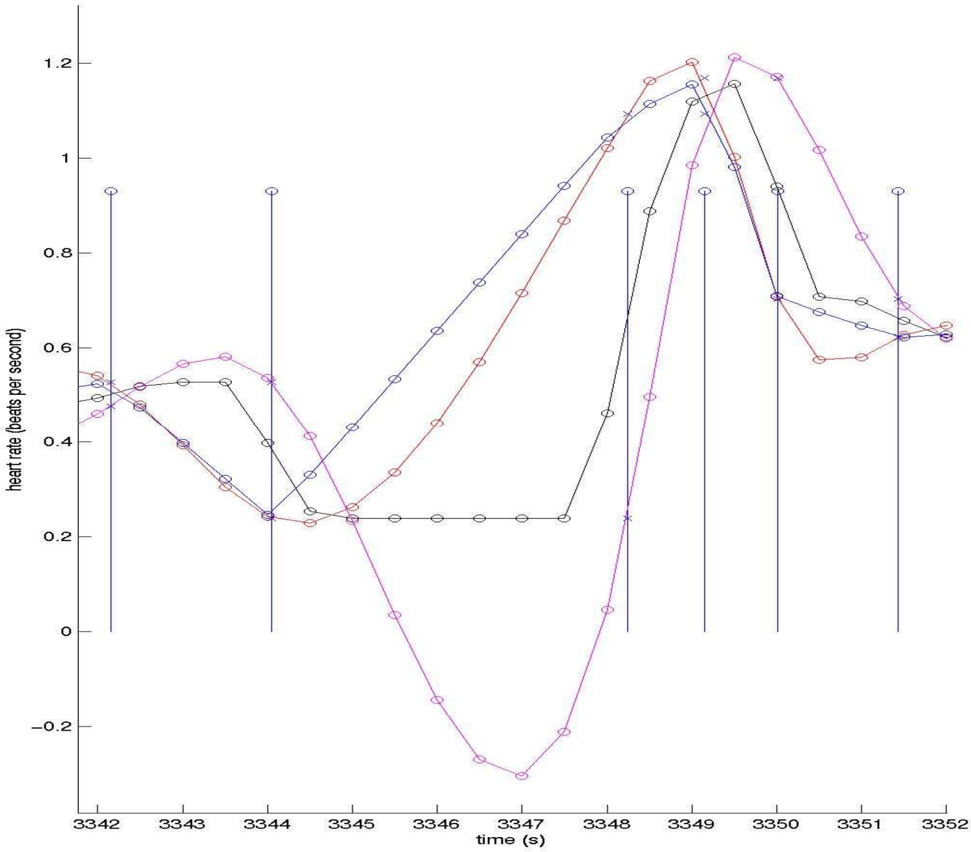

Figure 3: Heart rate changes during carotid sinus massage. *(Real data.) In this patient, carotid sinus massage triggered a four-second asystole. One of the two cubic splines methods failed to track heart rate, becoming negative for several data points. (In fact, if heart rate ever reaches zero, the patient must be dead since the next beat will not occur until infinite time passes.) In contrast, Berger* et al*'s algorithm produced an accurate heart rate of about 0.25 beats per second. Since a 3-second asystole is one of the criteria for a carotid sinus hypersensitivity diagnosis, a threshold of 0.333 beats per second could be used if necessary.*

The Wigner spectrum of stationary signals is simply the classical spectrum; clearly no advantage is to be gained in this case. Where SPWVD is truly valuable is in the analysis of signals whose spectra vary rapidly with time. Short-time Fourier transforms (STFTs) cannot accurately track changes in a signal's spectrum that occur over the course of a few seconds, which is a significant limitation for many biological signals. For example, the human nervous and cardiovascular systems are adept at modifying blood pressure and cardiac contraction behaviour in less than a few heart beats. An STFT with a 60-second moving window clearly misses such changes.

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3. HRV FROM THE LOMB PERIODOGRAM

An artificial tachogram was generated with the following characteristics, where LF and HF are abbreviations for low and high frequency respectively:

- Length of 512 seconds, with a mean heart rate of 60 beats per minute
- A sampling rate of 2 Hz (i.e., uneven sampling was never employed)
- No ectopic beats or artefacts
- A strong LF component concentrated in a 0.04-Hz band centred on 0.1 Hz
- A strong HF component concentrated in a 0.04-Hz band centred on 0.25 Hz
- The frequency smearing over each 0.04-Hz band was achieved by linearly modulating the LF and HF frequencies; hence, the signal was *non-stationary*.

In summary, the equation for this tachogram is given by:

$$HR = 60 + A_L \sin (w_L t) + A_H \sin (w_H t)$$

where $w_L = 0.08 + 0.04t/512$, $w_H = 0.23 + 0.04t/512$, $A_L = 0.1$ and $A_H = 0.08$.

The Lomb periodogram of this artificial tachogram matches the FFT (Fast Fourier Transform) spectrum (Figure 4). This is to be expected, for the Lomb equation reduces to a regular FFT process in the case of even sampling. Frequency-domain HRV was calculated on the central 256 seconds of the graph, using bounds of 0.04-0.15 Hz for LF and 0.15-0.40 Hz for HF. The result was was an LF/HF ratio of 1.56.

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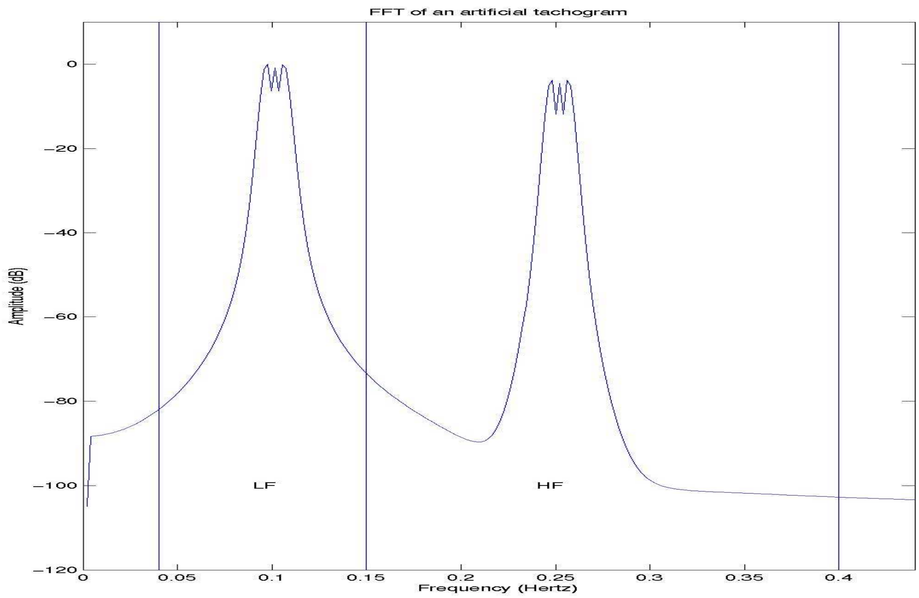
4. HRV FROM THE SPWVD

The tachogram used in Section 3 was next analysed in the time-frequency domain using the SPWVD. The frequency modulation is readily apparent in Figure 5. Summing the LF and HF contributions in the frequency axis led to an LF/HF ratio of 1.49. Fortunately, this is within 5% of the value obtained using the Lomb periodogram. Moreover, when this sort of comparative analysis was repeated for different tachograms, the SPWVD was found to perform consistently well (Table 1).

Tachogram parameters			Results		
LF range (Hz)	HF range (Hz)	$A_H : A_L$	Lomb LF/HF	SPWVD LF/HF	Per cent deviation
0.09-0.13	0.21-0.25	0.67	2.17	2.14	1.2%
0.08-0.12	0.23-0.27	0.8	1.56	1.49	4.5%
0.06-0.14	0.16-0.24	0.95	1.10	1.10	0.6%
0.08-0.12	0.28-0.32	1.1	0.821	0.801	2.4%
0.06-0.12	0.27-0.33	1.3	0.593	0.580	1.0%

Table 1: *Deviation of SPWVD LF/HF ratio from the classical method, for various artificial tachograms.*

(a)



(b)

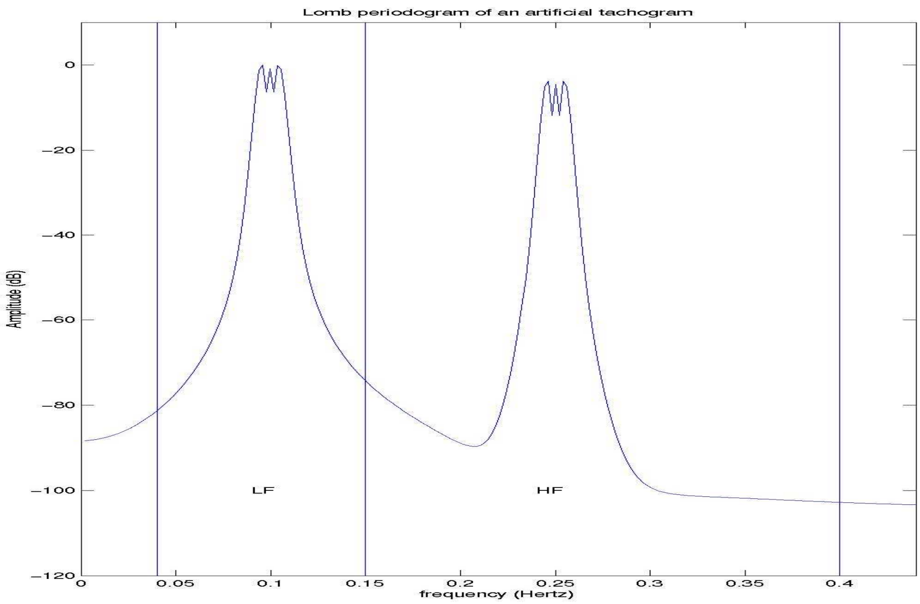


Figure 4: ***(a)*** *The FFT (Fast Fourier Transform) of the artificial tachogram being analysed.* ***(b)*** *The Lomb periodogram is identical. The three vertical lines mark the boundaries of the LF and HF regions.*

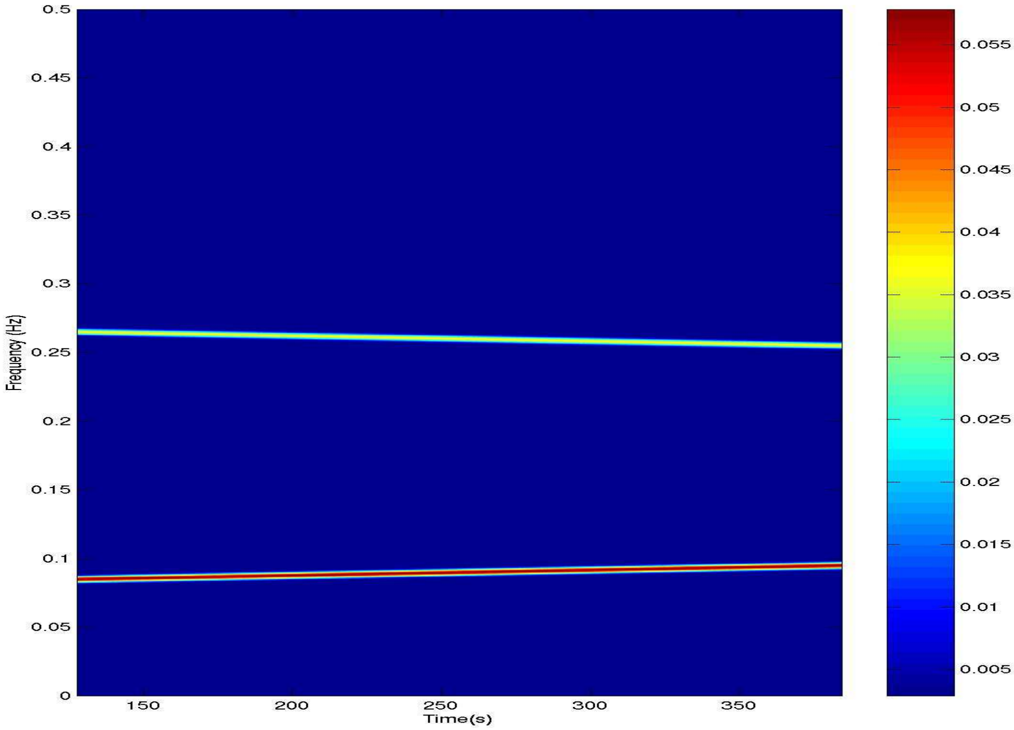


Figure 5: *SPWVD of an artificial tachogram. Note how the lower frequency rises with time and the higher frequency decreases with time; this is the frequency modulation intentionally imposed on the signal.*

5. DISCUSSION

It was shown that, on artificial data, the SPWVD can calculate LF/HF ratios as accurately as the classical method of doing so. The SPWVD is better than STFTs at tracking simple frequency components; for example, a linear chirp signal reduces to a string of Kronecker delta functions in the time-frequency plane (not shown here). The applicability of this *type* of precision is yet to be tested fully on patient data. A report summarizing these efforts will follow soon.

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